

A Technique for Avoiding Connection Errors in Computerized Impedance-Measuring Systems

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A Technique for Avoiding Connection Errors in Computerized Impedance-Measuring Systems

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Abstract—The technique described uses a series of impedance measurements with different lead combinations and a calculation to determine the impedance of an unknown in the presence of lead and loading impedances. In general, a four-terminal ac or dc measurement requires four leads, four switches, and a series of five two-terminal measurements. However, an ac bridge is shown that requires only two switches and three measurements. The impedance of the switches used to select the lead combinations has no effect on the measurement if it is constant and changes in switch resistance between closures can be avoided by choosing a measurement sequence that closes each switch only once.

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Most guarded four-terminal bridges are subject to errors caused by impedance to guard at the unknown end of the leads. A series of seven three-terminal measurements corrects for this type of error, which is particularly important for *in situ* measurements or for high-precision measurements on three-terminal standards.

The technique is particularly applicable to an automatic computerized device because two-terminal automatic bridges are substantially simpler than four-terminal bridges and because the speed of such a system and its computer can easily overcome the main disadvantages of the method—the necessity for several measurements and the calculation (which includes square roots). However, three two-terminal measurements and a simple calculation will measure a four-terminal impedance with a residual error that can be very small if the lead impedances are approximately equal. Thus the method may be practical for manual measurements as well.

INTRODUCTION

THE measurement technique described in this paper for avoiding connection errors is so simple that it is difficult to believe it has not been used previously. However, no references to it could be found. Metrology experts who were asked about it were unfamiliar with it. Therefore, even if it is not new but just lost somewhere in the literature, it should be revived for it would seem that it has particular application to modern automatic measuring systems that include a computer.

This multimeasurement technique for avoiding connection errors is particularly applicable to such systems because it simplifies their design appreciably and because they easily overcome the major disadvantages of the method.

The computer can correct for known sources of error internal to the bridge, but lead errors on a two-terminal bridge cannot be corrected for unless the impedance of the leads is known and remains constant. The usual method of measuring the lead impedance, shorting the leads together at the unknown, may depend critically on the resistance of the short. Also, the impedance of the actual connection to the unknown would be undetermined and variable. While a four-terminal bridge would remove lead errors, the additional adjustable components of a Kelvin bridge (the extra adjustable bridge arm and the "lead" and "yoke" balances) are expensive in an automatic bridge because they must be programmable and have additional logic to control them. Four-terminal ac bridges without additional adjustments have been described, but they do not remove lead errors entirely [1].

The technique requires a calculation that includes a square root; this is awkward to do by hand, especially if it is the square root of a complex number. Someone would have to write the program, but a digital computer could execute it quickly. The technique also requires several measurements to evaluate one unknown. Automatic bridges are fast; most of them are particularly fast if the differences between successive measurements are small, as they usually would be with this method.

There are several other multimeasurement techniques used for this type of measurement, such as the Mueller bridge, Smith's methods I and II, methods described by Kleven [2] and Riley [3], and probably others. However, this technique is distinctly different from these methods, particularly in that it can be used with any two- or four-terminal ac or dc bridge and with any bridge ratio.

FOUR-TERMINAL DC MEASUREMENTS

The technique is outlined in simplest form in Fig. 1. Here M_1 , M_2 , and M_3 are the results of three two-terminal measurements on the network shown. They are assumed to be corrected for all errors in the measuring device itself and, therefore, perfect. By using the formula given in Fig. 1,¹ the value of R_x can be calculated without error

¹ The positive value of all square-root terms should be used and added or subtracted as indicated.

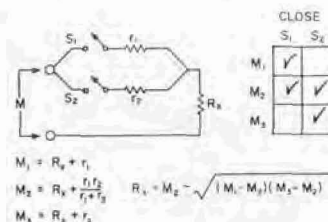


Fig. 1. Multimeasurement method, simplest form.

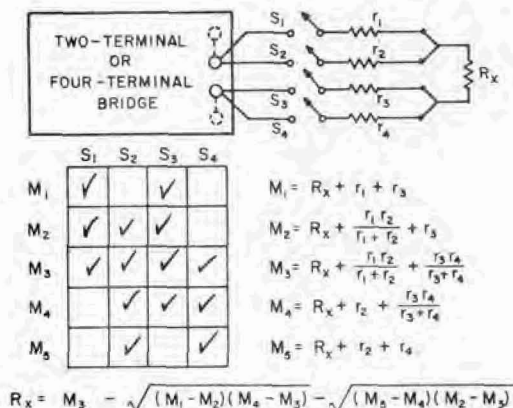


Fig. 2. Four-terminal resistance measurement using five two-terminal measurements.

from these three measurements, as long as the lead resistances r_1 and r_2 remain constant during the sequence of measurements. These resistances represent the total resistance in each branch including the resistances of the switch contacts. If the sequence shown is used, each switch is closed only once so that changes in contact resistance between closures are avoided.

The situation of Fig. 1 is generally impractical because the return connection to R_x would also have resistance. Fig. 2 shows a four-terminal measurement. Note that only two bridge terminals are required. Thus, this bridge may be a conventional Wheatstone bridge whose terminal resistance is known (and would be corrected for) or a four-terminal bridge connected as a two-terminal bridge as indicated by the dotted lines. A four-terminal connection of a Wheatstone bridge [3] or a Kelvin bridge could be used. In the latter, the yoke and lead resistances would only be those of the internal bridge wiring and the dotted connection shown, so that these adjustments should not be very critical and probably would need to be made only once. In an automatic system, the switches would be internal so that there would be four terminals or connectors.

In this circuit there are five undetermined quantities and five measurements are required to determine R_x exactly. Actually there are nine possible switch combinations, but four are redundant. There are several sets of five measurements that may be used. The sequence shown is one in which each switch is closed only once, as before, to avoid switch errors.

Recently, Pailthorp and Riley [4] suggested a set of five measurements that could determine all five quantities of

the four-terminal network of Fig. 2, using only two leads at a time. They measured $r_1 + r_2$ and $r_3 + r_4$ directly by appropriate connection. In their method, the resistance of the switches (or connections) could not be combined with the four lead resistances so that they introduced errors unless these were negligible.

Their method suggests the four-measurement sequence of Fig. 3 which has only one switch associated with each lead so that the switch and lead resistance may be combined as one quantity. This method has the advantage of a simpler calculation as well as one less measurement, but has the disadvantages that no sequence will give one closure per switch and that the measured values would differ greatly in most cases. Generally, successive measurements may be made more rapidly if they are approximately the same whether the bridge is manual or automatic.

It should be noted that these are true four-terminal measurements. If a four-terminal resistor were being measured, the resistances of leads internal to its structure would be included in the lead resistances shown. The calculated value of R_x would include only that resistance between the two lead junctions which is the definition of its four-terminal value.

FOUR-TERMINAL AC MEASUREMENTS

The technique is equally applicable to ac bridges of all types. The resistances in the formulas become impedances and therefore complex, but the procedure and formulas are the same.

The most precise ac bridges employ transformer ratio arms and may be two- or four-terminal. In some two-terminal bridges, the series impedance of the transformer (winding resistance and leakage inductance) appears in series with the unknown. In the bridge of Fig. 4 there are two equal windings on the unknown side of the bridge, each connected to one switch. Here the impedances z_1 and z_2 include the impedances of these windings so that their effect is removed by the correction terms of Fig. 2.

There will also be impedance in the connection of the bridge standard, Z_s , which may be determined and corrected for. Alternatively, Z_s may be connected by two windings and four switches as is Z_x , another set of measurements made, and further correction terms calculated.

The two-transformer bridge of Fig. 5 has the further advantage that one quantity can represent the total impedance of each loop, including the impedances of the switch, two connecting leads and two transformer windings. The simple three-measurement sequence and formula of Fig. 1 would be used. Again, Z_s also could be connected with two pairs of windings and two switches and more measurements and corrections made.

A remaining source of error in ac bridges is the mutual inductance between the leads that can appear effectively in series with the unknown. The position of the leads would be critical in very-low-impedance or high-frequency measurements, but their mutual inductance is constant if their position is fixed and therefore may be corrected for.

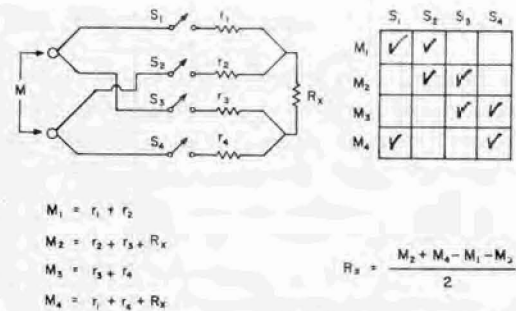


Fig. 3. Four-terminal measurement using four two-terminal measurements.

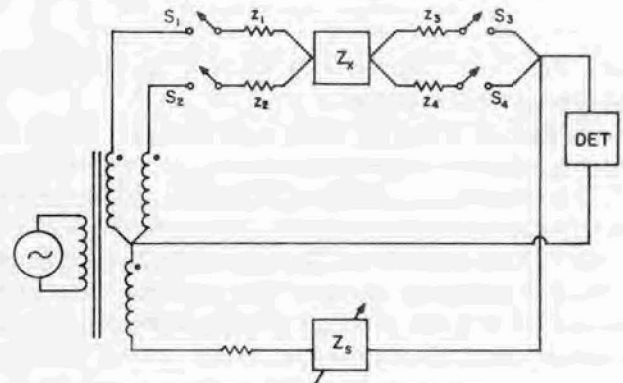


Fig. 4. Ac four-terminal measurement with winding impedances included in total lead impedance.

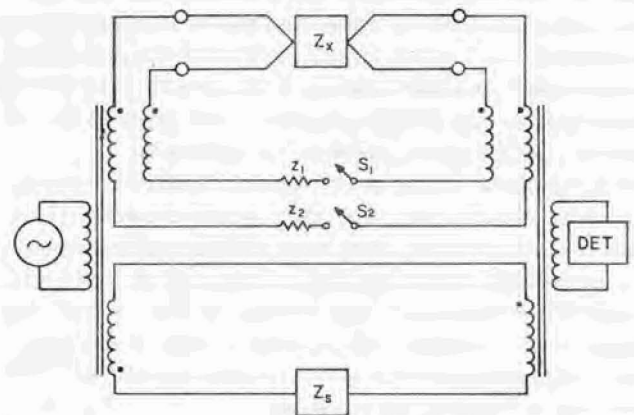


Fig. 5. Ac four-terminal measurement using three two-terminal measurements.

FIVE-TERMINAL MEASUREMENTS

A resistance bridge with a Wagner guard can make a direct three-terminal measurement on a three-terminal network, ignoring resistance from either unknown terminal to guard. However, if the leads have appreciable resistance, as in Fig. 6, not only will they cause errors by themselves, but if the shunt resistances (R_a and R_b) are at the unknown end of the leads, the divider action of the leads and the shunts will cause additional error terms. This type of error is important in *in situ* measurements of components connected in a network. It is also important in extremely precise measurements on three-terminal devices [5]. Most guarded four-terminal (or five-terminal) bridges will not

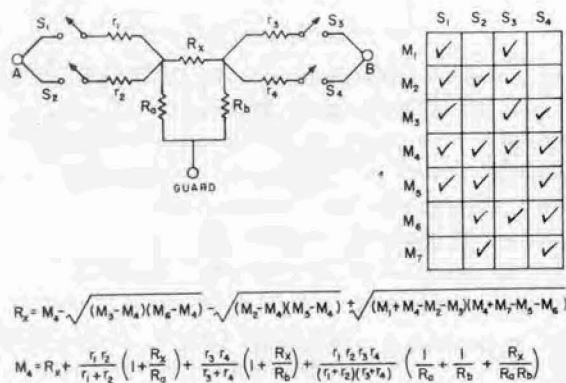


Fig. 6. Five-terminal measurement with loading at unknown.

remove this error.² However, a series of three-terminal measurements will remove it.

In the circuit of Fig. 6, there are seven undetermined quantities and seven measurements are required to obtain R_x . Unfortunately, no sequence of seven measurements gives only one closure per switch. The one shown requires that two switches close twice. The required calculation now has three square-root terms. The expression for one measurement is given to show its form, particularly the interaction terms. The others are easily derived from it.

If the shunt resistances R_a and R_b are connected together at the unknown and their junction connected to the bridge guard by one lead, the resistance of this lead can also cause error in some cases. One solution might be to use two guard leads, two more switches, and more measurements to determine R_x . (An exact formula has not been determined for this case.) Another solution is to remove the internal connections to the bridge guard and to bring them out separately to the guard point at the unknown.

Ac transformer-ratio-arm bridges are relatively immune from shunt loading so that they make accurate three-terminal measurements. However, they do not remove the error caused by shunt loading at the end of leads with appreciable impedance. The bridge of Fig. 7 and the measurement sequence and formula of Fig. 6 will give a result independent of this source of error as well as of the errors due to the leads themselves.

APPROXIMATE CORRECTIONS

While the technique described above would appear to be particularly suitable for automatic computer-controlled systems, it can also be used with manual bridges. A modern desk calculator would be helpful to make the calculations, but the complex ac calculations might still be different. However, in many applications a simplified formula can give the result with negligible error.

The three-measurement method outlined in Fig. 8 uses only three of the five measurements outlined in Fig. 2 and has a very simple correction term. The remaining error shown contains squared resistance-difference factors so that it can be very small if matched leads are used.

² Some active bridges or bridges with active guard circuits will avoid this error. See also [5].

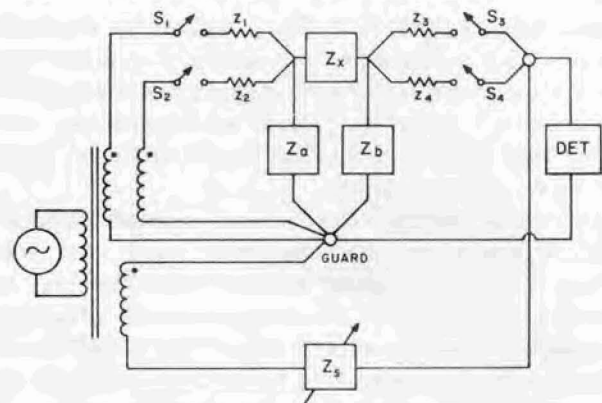


Fig. 7. Ac five-terminal measurement.

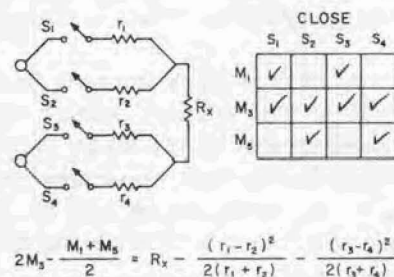


Fig. 8. Approximate four-terminal measurement.

This method would be useful in resistance thermometry where the lead resistances are rather high, but lead resistance differences can be small. One platinum resistance thermometer measured (Leeds and Northrup type-8163 with 8-ft leads) had lead resistances of about 1 Ω each, but resistance differences $r_1 - r_2$ and $r_3 - r_4$ of only 6 and 10 m Ω . This would give a residual error of about 35 $\mu\Omega$ or a little over 1 ppm. Moreover, if the switches (and leads to them from the bridge) have low or matched resistance, this error is mainly dependent on the thermometer being used. This quantity could be easily determined and noted. If this correction were also made, the accuracy of the method might be beyond the absolute accuracy of any thermometer or bridge available. For precise temperature difference measurements where extreme resolution is important, this residual error could be ignored since it would be almost constant and cancel out.

The simplified formula of Fig. 8 was arrived at by using the expression

$$(ab)^{1/2} = \frac{a+b}{2} \left[1 - \left(\frac{a-b}{a+b} \right)^2 \right]^{1/2} \simeq \frac{a+b}{2}, \quad a \simeq b. \quad (1)$$

By using this approximation for the full five-terminal case (Fig. 5),

$$z_x \simeq 4M_4 + \frac{M_1 + M_7}{2} - M_2 - M_3 - M_5 - M_6 \quad (2)$$

with error terms all containing $(r_1 - r_2)^2$ and $(r_3 - r_4)^2$ factors so that if the lead impedances are well balanced the

errors are small. Even better error expressions may be obtained by expanding the square root into a power series and using as many terms as required.

CONCLUSIONS

The technique described for making multiterminal impedance measurements would appear to be particularly useful in computerized impedance-measurement systems. It could be applied to manual measurements as well.

Several measurements were made to check the formulas using rather exaggerated values of lead and shunting impedance. No actual computerized system was used so that this application is pure conjecture.

Other areas of possible application would be *in situ* measurements, precision measurements on three-terminal standards, high-frequency measurements, and perhaps the measurement of voltage and current.

Author's Note

R. M. Pailthorp of Electro-Scientific Industries points out that the method credited to J. C. Riley and him [4] was described by G. F. C. Searle in 1911. ("On resistances with current and potential terminals," *The Electrician*, no. 1715, March 31, 1911, reprints available as

Technical Article TA-12 from ESI.) This paper also describes the method of Fig. 3.

Pailthorp also comments that the method of Fig. 3 has the important advantage that the zero resistance of a two-terminal bridge used to make this set of measurements would not affect the calculated result. He also mentions that this method has the disadvantage of having the open-circuit impedance of the unused switches shunting the unknown when it is included in the circuit (M_2 and M_4). This would be important for high-impedance measurements.

REFERENCES

- [1] H. P. Hall, "Four-terminal equal-power, transformer-ratio-arm bridge," *IEEE Trans. Instrum. Meas.*, vol. IM-19, Nov. 1970, pp. 308-311. Residual lead errors are compared for seven bridges.
- [2] L. A. Kleven, "Resistance measuring method and apparatus having means for alternating connecting unknown resistor to different arms of bridge," U.S. Patent 3 461 383, Aug. 12, 1969.
- [3] J. C. Riley, "Four terminal measurements with a Wheatstone bridge," *Electro-Sci. Ind. Tech. Rep.*, no. 19.
- [4] R. M. Pailthorp and J. C. Riley, "Precision measurement of resistor networks," *Proc. 1971 Electron. Components Conf.* Washington, D.C., May 1971.
- [5] R. D. Cutkosky, "Techniques for comparing four-terminal-pair admittance standards," *J. Res. Nat. Bur. Stand., Sect. C*, vol. 74, no. 3-4, July/Dec. 1970.